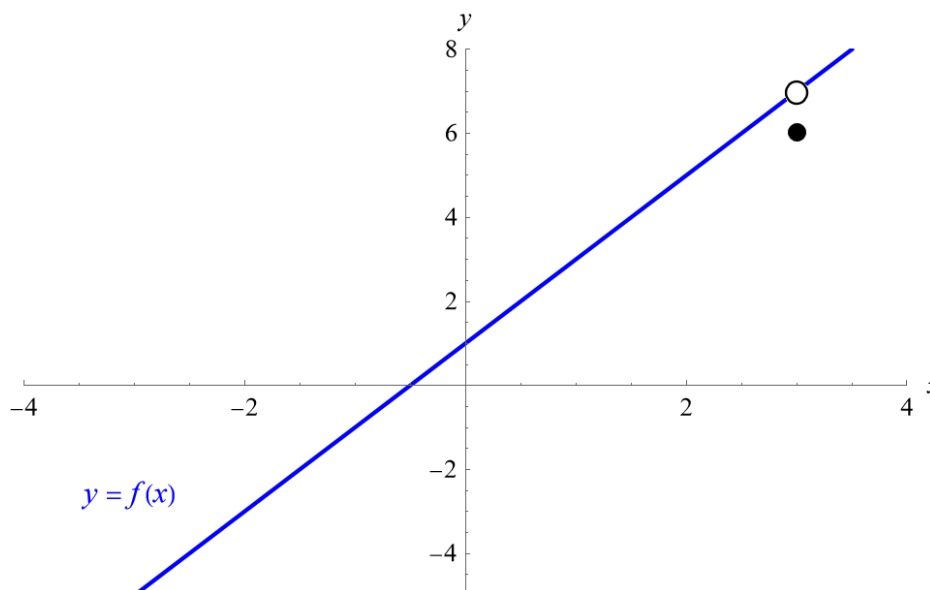## Exercise 22

Explain why the function is discontinuous at the given number a. Sketch the graph of the function.

$$f(x) = \begin{cases} \dfrac{2x^2 - 5x - 3}{x - 3} & \text{if } x \neq 3 \\[3mm] 6 & \text{if } x = 3 \end{cases} \qquad a = 3$$

Solution

A graph of the function versus x is shown below.

The function is discontinuous at $x = 3$ because although the left-hand and right-hand limits are both equal to 7 there, they are not equal to the value of the function there, which is 6.

$$\begin{aligned} \lim_{x \to 3} f(x) &= \lim_{x \to 3} \frac{2x^2 - 5x - 3}{x - 3} \\[2mm] &= \lim_{x \to 3} \frac{(2x + 1)(x - 3)}{x - 3} \\[2mm] &= \lim_{x \to 3} (2x + 1) \\[2mm] &= 2(3) + 1 \\[2mm] &= 7 \\[2mm] &\neq f(3) = 6 \end{aligned}$$